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SPLIT DELIVERY ROUTING

by

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ABSTRACT

This paper examines a relaxed version of the generic vehicle routing problem. In this version, a delivery to a demand point can be split between any number of vehicles. The problem remains a hard problem and subsequently the vehicle routes are constructed using heuristic rules. The solution scheme allowing for split deliveries is compared with a similar solution scheme in which no split deliveries are allowed. The comparison is conducted on six sets of 30 problems each. Each problem set is characterized by different demand parameters for the fixed set of 75 demand points. For very small demands (up to 10% of vehicle's capacity) no significant difference in solutions is evident for both solution schemes. For the other five problem sets for which point demand exceeds 10% of vehicle's capacity, very significant cost savings are realized when allowing split deliveries. The savings are significant both in the total distance and the number of vehicles required. The vehicles' routes constructed by our procedure, tend to cover geographic zones and retain some properties of optimal solutions.

Subject Classification : 483 vehicle routing, 837 multiple visits.

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RÉSUMÉ

Nous examinons une version moins contrainte du problème de tournées ("vehicle routing problem"). Dans cette version, plus d'un véhicule peut effectuer une livraison en un point pour en satisfaire la demande. Une heuristique est présentée et les résultats composés avec une solution similaire où chaque demande doit être répondue par un seul véhicule. La comparaison est faite avec six ensembles de 30 problèmes chacun ayant les mêmes 75 points de demande. Pour de très petites demandes (jusqu'à 10% de la capacité du véhicule) il n'y a pas d'amélioration significative. Mais, pour des demandes plus élevées, on observe des gains significatifs tant au point de vue de la distance parcourue que du nombre de véhicules requis.

1. Introduction

Let there be n demand points in a given area (say set N). Assume that a daily demand of a specific commodity has to be delivered daily to each of the demand points. This daily demand at each point is known and is usually different at the different points. Denote by d_i ; i = 1,2,3,...,n the demand at point i. Assume that the deliveries to this set N originate from one central location, the depot, and that the total daily demand, $(\sum_{i=1}^{n} d_i)$, is always available at the center. In addition, assume that a fleet of vehicles with individual capacities is dedicated to service this set N of demand points. The direct travel distance between the demand points including the depot is known, and is denoted by C_{ij} for points i and j. An objective for an efficient distribution activity in this setting, is the total distance minimization of the delivery routes assigned to the fleet of vehicles. The problem of constructing these routes with the additional requirement that no demand point is to be serviced by more than one vehicle, is generally known as the capacitated Vehicle Routing Problem or in short the VRP. (See for example the following references: Bodin et al. (1983), Christofides (1985), Dror and Trudeau (1986), Laporte, Mercure, Norbert (1986).)

Sometimes, when respecting only the vehicle capacity constraint, an individual route can be constructed with a total distance exceeding an acceptable length. It can be prevented by adding a length constraint on the vehicles' routes to the set of requirements. This version is referred to as the 'time' constrained VRP. In what follows we ignore the 'time' restriction on the individual routes.

In this paper we concentrate on the cost (distance) implications for the solution of the VRP, which can be attributed to the service requirement of no more than one vehicle visiting one demand point. The problem setting in which one is allowed to split the service of a demand point between more than one vehicle is referred to in this paper as the Split Delivery Vehicle Routing Problem and denoted by SDVRP.

We assume for convenience throughout this paper that no individual customer demand is greater than the capacity of the smallest vehicle. This restriction can be dropped with minor modification in the subsequent presentation.

It is very simple to demonstrate potential cost (distance) implications of allowing split deliveries to the demand points. The following example serves as one illustration:

Example 1. : Assume three demand points with $d_1 = 3, d_2 = 4, d_3 = 3$, the distances between the points including the depot is: $C_{0i} = 10$, for i = 1,2,3; $C_{12} = 5 = C_{23}$, and $C_{13} = 10$. The vehicles at the depot have the same capacity of 5 units.

The solution to the above VRP (no splits allowed) has a total cost of 60 and requires three vehicles. The SDVRP solution to the above problem costs only 50 and requires only two vehicles. (Figures 1a and 1b respectively.)

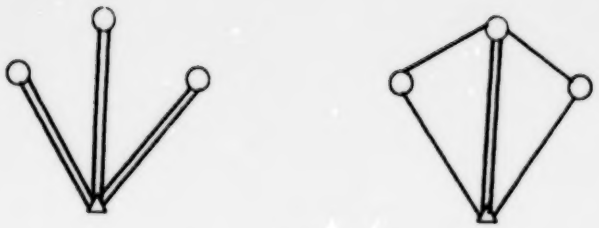

Figure 1a : The VRP solution. **Figure 1b** : The SDVRP solution.

We begin the presentation (in Section 2.) with the mathematical formulation of the split delivery vehicle routing problem - (SDVRP) and relate it to the 'generic' vehicle routing problem. Complexity results are derived for the SDVRP and conjectured for an embedded problem. In Section 3 some structural properties of the optimal SDVRP solutions are indicated. In Section 4 we introduce two new heuristic procedures: k-Split Interchange and Route Addition, specific to the SDVRP which when added to any VRP solution scheme allow for split deliveries. The Main Algorithm is presented in Section 5 and the extensive computational experiments are presented in Section 6. A summary and conclusions are given in Section 7. Throughout this paper the terms cost and distance are used interchangebly.

2. Mathematical Formulation of the SDVRP - (P) and Related Problems

Notation

C_{ij} = The distance ('cost') between demand points i and j. (We assume in this paper a symmetric distance metric between all points unless explicitly stated to the contrary.)

d_i = The daily demand at point i.

Q_v = The capacity of the v^{th} vehicle.

x_{ij}^v = 1 if the v^{th} vehicle travels directly from point i to point j, and $x_{ij}^v = 0$ otherwise.

y_{iv} = The fraction of the i^{th} point demand delivered by the v^{th} vehicle.

NV = The number of vehicles in the fleet.

The point 0 denotes the depot.

$$\text{Minimize } z_s = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{v=1}^{NV} C_{ij} x_{ij}^v \qquad (1)$$

Subject to

$$\sum_{v=1}^{NV} \sum_{i=0}^{n} x_{ij}^v \geq 1 \qquad \text{for j} = 0,1,\ldots,n \qquad (2)$$

$$\sum_{i=0}^{n} x_{ip}^{v} - \sum_{j=0}^{n} x_{pj}^{v} = 0 \qquad \text{for p = 0,1,...,n; v = 1,2,...,NV} \qquad (3)$$

$$\sum_{v=1}^{NV} y_{iv} = 1 \qquad \text{for i = 1,2,...,n} \qquad (4)$$

$$\sum_{i=1}^{n} d_i y_{iv} \leq Q_v \qquad \text{for v = 1,2,...,NV} \qquad (5)$$

$$y_{iv} \leq \sum_{j=0}^{n} x_{ji}^{v} \qquad \text{for i = 1,2,...,n; v = 1,2,...,NV} \qquad (6)$$

$$\mathbf{X} \in S \qquad (7)$$

$$x_{ij}^{v} = 0,1 \qquad \text{for i = 0,1,...,n; j = 0,1,...,n; v = 1,2,...,NV} \qquad (8)$$

$$y_{iv} \geq 0 \qquad \text{for i = 1,2,...,n; v = 1,2,...,NV} \qquad (9)$$

The above mathematical formulation expressed by (1) - (9) will be referred to thereafter as the (P) problem.

Equation (1) in (P) expressess the distance traveled by the fleet of vehicles. Equation (2) insures that each demand point will receive at least one delivery. Constraints (3) are the conservation of flow constraints forcing the vehicle which enters a demand point (including the depot) to leave the demand point. Constraints (4) insure that each point will receive its full demand. Constraints (5) insure that the delivery assignment for a vehicle does not exceed the vehicle's capacity. Demand point i can be serviced only by a vehicle passing through that point. This is inforced by constraints (6). Constraints (7) are the subtour eliminating constraints whose function is to insure that the vehicle routes always start and end at the depot. Constraints (8) are the binary value constraints for the set of the routing variables x_{ij}^{v}. Constraints (9) are the nonnegativity constraints on the fractional variables y_{iv}.

Note that in (P) constraints (2) are redundant and can be obtained simply by summing constraints (6) over v and using constraint (4).

For the sake of completeness and comparison we present the mathematical model of the VRP. Similar or identical formulations of this model can be found in many references. For example: Bodin et al. (1983), Christofides (1985), Fisher, Jaikumar (1981), Gavish and Graves (1982), Golden, Magnanti, Nguyen (1977), Laporte, Norbert, Desrochers (1985), and many more.

2.1 Mathematical Formulation of the VRP - (P1)

$$\text{Minimize } z_f = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{v=1}^{NV} C_{ij} x_{ij}^{v} \qquad (10)$$

Subject to

$$\sum_{v=1}^{NV} \sum_{i=0}^{n} x_{ij}^{v} = 1 \qquad \text{for j = 0,1,...,n} \qquad (11)$$

$$\sum_{i=0}^{n} x_{ip}^{v} - \sum_{j=0}^{n} x_{pj}^{v} = 0 \qquad \text{for v = 1,2,...,NV; p = 0,1,...,n} \tag{12}$$

$$X \in S \tag{13}$$

$$\sum_{j=1}^{n} \sum_{i=1}^{n} d_i x_{ij}^{v} \leq Q_v \qquad \text{for v = 1,2,...,NV} \tag{14}$$

$$x_{ij}^{v} = 0,1 \qquad \text{for i = 0,1,...,n; j = 0,1,...,n; v = 1,2,...,NV} \tag{15}$$

The above mathematical formulation expressed by (10) - (15) will be referred thereafter as the (P1) problem.

When comparing the two problems (P) and (P1), the similarities and the differences are almost self explanatory. Clearly, the objective function remains the same. Constraint (2) changes from $' \geq 1'$ in (P) to $' = 1'$ in (P1) (constraint (11)). The vehicle routes in (P) are specified not only by their spacial configuration, but also by the quantity delivered at each stop. This is expressed in (P) by the constraints (4), (5), and (6). In (P1) the routes are fully specified by their spacial configurations only since the quantities delivered at each stop are prefixed by the problem parameters: the d_i's. The capacity restriction on a route is expressed directly through the binary routing variables x_{ij}^{v} (constraint (14)).

Since in the SDVRP, in contrast to the VRP, one is no longer forced to construct routes which deliver the full demand requirement at each point on the route, the SDVRP can be considered a relaxation of the VRP. The route planner-analyst has more freedom when constructing the routes. Is the SDVRP an 'easier' problem to solve ? This question is answered by the following theorem:

Theorem 1 : The SDVRP is an NP-complete problem.

Proof: It is clear that the SDVRP is in NP. In case that $min_v Q_v \geq \sum_{i=1}^{n} d_i$ and the $\{C_{ij}\}$ matrix satisfies the triangular inequality the SDVRP reduces to the celebrated Traveling Salesman Problem-(TSP). Thus, by reduction, (see Garey, Johnson, 1979 and Lenstra, Rinnooy Kan, 1981) the SDVRP is NP-complete.

The VRP is, by now, clearly also an NP-complete problem. (In fact, exactly the same proof as for the SDVRP holds for the VRP.) At this point, we formalize the fact that the SDVRP is a relaxation of the corresponding VRP •

Theorem 2 : Let the pair (x_s^*, y_s^*) be an optimal solution for (P) with the objective value z_s^*; and let x_f^* be the optimal solution to the corresponding (P1) with objective value z_f^*, then $z_s^* \leq z_f^*$.

Proof: It is clear that $z_s^* \not> z_f^*$ for any problem instance since x_f^* is a feasible solution of the corresponding (P).

In order to prove that $z_s^* \leq z_f^*$ it is sufficient to find one problem instance for

which $z_s^* < z_f^*$. See example 1 •

The (P) formulation with the binary constraints on the x_{ij}^v variables (constraints (8)) removed is the linear programming relaxation of (P). Denote that LP formulation as the (P_{lp}) problem and its optimal solution value by z_{lp}^*. The obvious relation is: $z_{lp}^* \leq z_s^* \leq z_f^*$.

Since the SDVRP is an NP-complete problem, one is inclined to examine the (P) formulation to search for computationally attractive subproblems in order to construct a 'good' heuristic algorithm for (P). In the section below we discuss a subproblem of the SDVRP and conjucture a complexity result.

2.2 The Reduced Problem - (RP)

$$\text{Minimize } z_r = \sum_{i=0}^{n} \sum_{j=0}^{n} C_{ij} \sum_{v=1}^{NV} x_{ij}^v \tag{16}$$

Subject to

$$\sum_{i=0}^{n} x_{ij}^v - y_{jv} \geq 0 \quad \text{for j = 1,2,...,n; v = 1,2,...,NV} \tag{17}$$

$$\sum_{v=1}^{NV} y_{iv} = 1 \quad \text{for i = 1,2,...,n} \tag{18}$$

$$\sum_{i=1}^{n} d_i y_{iv} \leq Q_v \quad \text{for v = 1,2,...,NV} \tag{19}$$

$$y_{iv} \geq 0 \quad \text{for i = 1,2,...,n; v = 1,2,....NV} \tag{20}$$

$$x_{ij}^v = 0,1 \quad \text{for i = 0,1,...,n; j = 0,1,...,n; v = 1,2,...,NV} \tag{21}$$

Denote the optimal solution value for the (RP) problem by z_r^*. Note that $z_r^* \leq z_s^*$ since the (RP) is a (proper) relaxation of the (P) problem. On the other hand, in general $z_{lp}^* \not\leq z_r^*$, since one can constuct problem instances for which $z_{lp}^* > z_r^*$.

The (RP) problem is essentially the (P) problem with the subtour elimination constraints, (7), dropped. If in the (P1) problem (no splits allowed), the subtour elimination constraints are dropped, it is easy to prove that the problem still remains NP-complete by virtue of reduction to the generalized assignment problem which is NP-complete (see Fisher 1981, Fisher and Jaikumar 1981, Garey and Johnson 1979). The above argument is less clear in the case of the (P) problem since its knapsack constraints (19) have continuous (y_{iv}) rather than binary variables. If there were a 'fast' (polynomial) solution technique for the (RP), one would be tempted to incorporate it into a heuristic scheme for the (P) problem. An optimal solution for the (RP) would tend to group the demand points (split and not split) into suitable subsets whose total demand does not exceed the capacity of the assigned vehicle. Unfortunately, we

Unfortunately, we were unable to find such 'fast' solution method for the (RP). Moreover, we conjucture that the (RP) is an NP-complete problem.

Conjucture 1 : The (RP) problem is NP-complete. (Assume that more than one vehicle is required.

3. Properties of the Optimal SDVRP Solutions

Next, we prove that in the optimal solution of the SDVRP, when the $\{C_{ij}\}$ matrix satisfies the triangular inequality, two demand points whose supplied demand is split among a number of routes do not belong to the same two routes.

Theorem 3 : If the $\{C_{ij}\}$ matrix satisfies the triangular inequality then no two routes in the optimal solution of (P) can have more than one split demand point in common.

Proof : For simplicity's sake, assume two demand points p and q receive their full demand d_p and d_q in a split manner by the same two routes 1 and 2. The quantity delivered to point p by route 1 is d_{1p} and by route 2 d_{2p}. Similarly define d_{1q} and d_{2q}. $(d_{1p} + d_{2p} = d_p; d_{1q} + d_{2q} = d_q)$. This is illustrated in Figure 2.

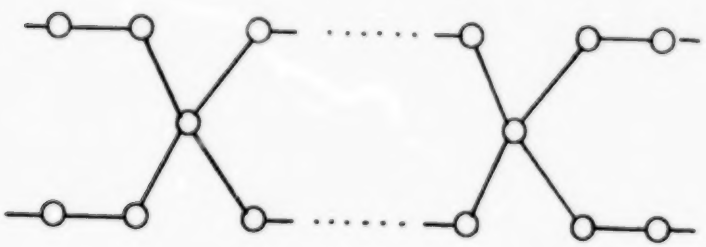

Figure 2 : The two route two split example.

It is clear (employing the triangular inequality of the C_{ij}'s) that eliminating one vehicle's visit to one of the two demand points p and q, results in decreasing (nonincreasing) the value of the objective. One is left with proving that the four deliveries $d_{1p}, d_{1q}, d_{2p}, d_{2q}$ could be exchanged by three deliveries by the same two vehicles without violating the vehicles' capacity constraints.

Let $d_{min} = Min\{d_{p1}, d_{q1}, d_{p2}, d_{q2}\}$

and assume for simplicity that $d_{min} = d_{p1}$

Obtain the new quantities delivered to demand points p and q by routes 1 and 2 as follows:

$d_{q1}^{new} = d_{q1} + d_{p1}$

$d_{p2}^{new} = d_{p2} + d_{p1}$

$d_{q2}^{new} = d_{q2} - d_{p1}$

$d_{p1}^{new} = 0$

The capacities of the vehicles 1 and 2 are not violated and the total quantities delivered to p and q remain the same as before •

Next, we present a generalization of Theorem 3 for which we need the following definition:

Definition : Given k demand points $p_1, p_2, ..., p_k$ and k routes. Route 1 includes the points p_1, p_2, route 2 includes the points p_2, p_3, ..., route k-1 includes points p_{k-1}, p_k, and route k includes points p_k, p_1. (This implies that the points $p_1, p_2, ..., p_k$ receive split deliveries by the k respective routes and possibly other routes) We call this subset of demand points $\{p_i\}_{i=1}^{k}$ the **k-split cycle**.

Theorem 4 : If the $\{C_{ij}\}$ matrix satisfies the triangular inequality then there is no k-split cycle (for any k) in the optimal solution to the problem (P).

Proof : The proof follows the same argument as in the proof of Theorem 3.

A number of studies were conducted in the attempt to solve exactly the VRP-(P1) problem. An abbreviated list of such studies includes Christofides, Mingozzi, Toth (1980), Christofides, Mingozzi, Toth (1981), Laporte, Norbert, Desrochers (1985), Laporte, Mercure, Norbert (1986). Without elaborating on the enumerative solution methods proposed in those studies, one major conclusion is that problem instances in which the number of demand points exceeds 60 are prohibitively expensive in terms of computer resources (time and memory). Whether it remains true in the case of the SDVRP remains an open problem for further investigation. This paper, in what follows, examines a heuristic solution scheme for the SDVRP and compares it with a 'successful' (see Dror, Ball, and Golden 1985/6, Dror and Levy 1986) similar scheme for the VRP. This comparison has a twofold purpose: (i) To assess the impact (in terms of distance (cost) and in the number of vehicles) of relaxing the one delivery to a demand point constraint. (ii) To computationally test the proposed solution scheme for the SDVRP. Since the main objective was (i), a well known test problem of 75 demand points was selected (from Eilon, Watson-Gandy, Christofides 1971, see also Dror and Trudeau 1986) and the problem's parameters were varied. All together 180 problems were solved. This computational experiment is described in detail in Section 6.

4. Heuristic Procedures for the SDVRP

Note: The description of the proposed heuristic scheme for the SDVRP assumes at the outset that a feasible solution to the corresponding VRP is given. No preas-

sumption on the 'quality' of that feasible VRP solution is necessary.

Given NV vehicle routes: one for each vehicle. The vehicles' capacities are $Q_1, Q_2, ..., Q_{NV}$. Each vehicle route (say the i^{th}) consists of an ordered subset of demand points denoted by D_i. The order of the demand points in D_i implies the routing sequence for the vehicle i. At the outset, since no split deliveries are allowed in the VRP solution, the D_i's are mutually exclusive subsets of demand points (except for the depot). (I.e. $D_i \bigcap D_j = \{0\}$ for all $i \neq j$, and $\bigcup_{i=1}^{NV} D_i = N$).

$$\text{Let } s_v^0 = Q_v - \sum_{j=1}^{n} \sum_{i=1}^{n} d_i x_{ij}^v, \quad \text{for v} = 1,2,....,\text{NV} \qquad (22)$$

s_v^0 denotes the spare capacity of the vehicle v on its initial route.

In terms of the SDVRP solution, since initially the y_{iv} variables are either 0 or 1 then:

$$s_v^0 = Q_v - \sum_{i=1}^{n} d_i y_{iv} = s_v, \quad \text{for v} = 1,2,....,\text{NV} \qquad (23)$$

In general, for a given SDVRP solution, if there are K nonempty vehicle routes , then s_v (without a superscript) denotes the spare capacity of vehicle v on its route.

4.1 k-Split Interchange:

At this point we introduce what we call a '2-split interchange'. This is generalized later to k-split interchange for $k \geq 3$. For simplicity, let the three routes taking part in the interchange be numbered 1,2, and 3.

Assume that $s_1, s_2 > 0$ and that there is a demand point on route 3, say point p, with demand delivered by route 3 of d_{p3} for which the following holds:

$$s_1 + s_2 \geq d_p \qquad (24)$$

Let i_1, j_1 be two consecutive points on route 1, and i_2, j_2 two consecutive points on route 2. Let b_p and a_p be the two points which come immediately *before* and *after* point p on route 3. Then, the **savings** term obtained if the demand d_{p3} is split between the routes 1 and 2 (i.e., point p removed from route 3 and set between i_1, j_1 , and i_2, j_2 respectively) is as follows:

$$SAV_3(p) = C_{i_1 j_1} + C_{i_2 j_2} + C_{b_p p} + C_{p a_p} - C_{i_1 p} - C_{p j_1} - C_{i_2 p} - C_{p j_2} - C_{b_p a_p} \qquad (25)$$

If, for some demand point p, condition (24) is satisfied and the corresponding savings term is positive, then the total routing cost can be reduced by the value of the savings term by spliting the demand d_{p3} between the routes 1 and 2.

Note that it never pays (because of the triangular inequality of the $\{C_{ij}\}$'s) to split the demand delivered to point p between the route 3 and one of the other routes 1 or 2.

The generalization from a 2-split interchange to k-split interchange for $k \geq 3$ is straightforward as follows:

$$SAV_k(p) = \sum_{l=1}^{k}(C_{i_l j_l} - C_{i_l p} - C_{p j_l}) + C_{b_1 p} + C_{p a_p} - C_{b_1 a_p} \qquad (26)$$

If $s_1 + s_2 + ... + s_k \geq d_{p(k+1)}$ and the term $SAV_k(p)$ is positive, then the demand point p can be eliminated from route k+1 and the total routing cost reduced.

In case the demand delivered to point p is already split between a number of routes, then the k-Split Interchange procedure examines all split configurations for the point p, and not only additional splits of delivery on one route leaving the other split deliveries unchanged. This is accomplished by: (i) removing the point p from all the split routes and computing the distance saved, (ii) each combination of routes for which the total spare vehicles capacity is greater or equal to d_p is examined for possible split delivery to point p inserting the point p in the least costly manner on those routes. The same equation (26) ($SAV_k(p)$) is used in this process to compute whether a different split configuration for point p is in order.

So far, nothing was presented to indicate how to set the division of the demand point p was receiving from the $(k+1)^{th}$ vehicle between the k vehicles in case more than one division is possible. The approach adopted in this study follows the 'best fit' notion described in [11] for the case of best fit bin packing algorithm.

4.2 Route Addition :

In some cases, addition of a route which eliminates a split delivery to a demand node might also reduce the total routing cost. Example in Figure 3a and 3b.

Figure 3a : 2-routes with one split.

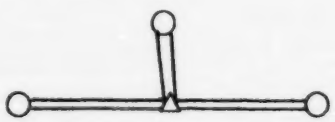

Figure 3b : 3-route and no split.

In the example above, the demand point 2 is split between two routes (see Figure 3a) with total routing cost of $C_{01} + C_{12} + C_{20} + C_{02} + C_{23} + C_{30}$. With the addition of one route the total cost is: $C_{01} + C_{10} + C_{02} + C_{20} + C_{03} + C_{30}$. The change is: $C_{12} + C_{23} - C_{10} - C_{30}$.

In case a demand point p is split between two routes 1 and 2, in order to examine whether adding one route which eliminates that split and results in reduced total distance, we check all (9) different possible route configurations with the following properties: (i) preserve the principle route segments (from the depot to a point before p and from point after p to the deopt), (ii) for each route not visiting point p we link the two principle segments. For the two routes there are 4 route segments

in regards to the demand point p. For route 1 the first segment is from the depot (point 0) to b_{1p} - point before p on route 1. The second segment is from a_{1p} - point after p on route 1, to the depot. Similarly for route 2. A new route, say route 3, which eliminates the split can be formed as follows:

Route Configurations			
No.	Route 1	Route 2	Route 3
1.	$0, b_{1p}, a_{1p}, 0$	$0, b_{2p}, a_{2p}, 0$	0,p,0
2.	$0, a_{1p}, 0$	$0, b_{2p}, a_{2p}, 0$	$0, b_{1p}, p, 0$
3.	$0, b_{1p}, 0$	$0, b_{2p}, a_{2p}, 0$	$0, p, a_{1p}, 0$
4.	$0, b_{1p}, a_{1p}, 0$	$0, a_{2p}, 0$	$0, b_{2p}, p, 0$
5.	$0, b_{1p}, a_{1p}, 0$	$0, b_{2p}, 0$	$0, p, a_{2p}, 0$
6.	$0, b_{1p}, 0$	$0, b_{2p}, 0$	$0, a_{1p}, p, a_{2p}, 0$
7.	$0, a_{1p}, 0$	$0, a_{2p}, 0$	$0, b_{1p}, p, b_{2p}, 0$
8.	$0, a_{1p}, 0$	$0, b_{2p}, 0$	$0, b_{1p}, p, a_{2p}, 0$
9.	$0, b_{1p}, 0$	$0, a_{2p}, 0$	$0, a_{1p}, p, b_{2p}, 0$

Table I : Route Addition configurations for 2-split delivery.

Alltogether, for this case of point p split between routes 1 and 2, there are $\binom{4}{2} - 2 + 4 + 1 = 9$ different configurations of single route addition which eliminates the split delivery to point p.

In general, in case of k-split for point p, the number of different configurations for a single route addition which eliminates such a split delivery is: $\binom{2k}{2} - k + 2k + 1 = 2k^2 + 1$.

The Route Addition procedure we developed, is restricted in the sense that it does not go beyond 3-split configurations when considering a route addition. In case a point is split between more than 3 routes, the Route Addition procedure examines each instance of 2 and 3 routes that visit the point and analyse 9 and 19 configurations respectively.

5. The Main Algorithm

First, we describe our two-stage Main Algorithm which constructs for a given problem instance both, the VRP solution in its first stage, and the SDVRP solution in its second stage. In addition, we notice that this algorithm preserves some characteristics of the optimal solution.

The Main Algorithm consists of five subroutines interconnected by conditional statements. The first subroutine is the Initial Route Generator, the second subroutine is the Node Interchange Algorithm , the third one is a Route Improvement Algorithm,

the fourth is the k-Split Interchange. and the fifth subroutine is the Route Addition Algorithm.

The first subroutine is described in detail in Bodin et al. (1983), and is a variant of the well known Clark and Wright algorithm for the initial route construction. The Node Interchange Algorithm is described in detail in Dror and Levy (1986) and is based on one-node and two-node swaps in between routes. The Route Improvement Algorithm is a version of the weel known 2-opt procedure and serves as post processor after each route construction. The combination of the first three subroutines proved very successfull in constructing efficient vehicle routes as demonstrated in Dror and Levy (1986), and Dror, Ball, and Golden (1985/6). The k-Split Interchange subroutine as well as the Route Addition Algorithm were described in the previous section.

The Main Algorithm

Stage 1 : (Input: Problem Instance)

Initial Route Generation.

Route Improvement.

Node Interchange Algorithm

Route Improvement.

Stage 2 : (Input: A Feasible VRP Solution)

k-Split Interchange.

Route Addition.

Node Interchange and Route Improvement.

Each of the stages (1 and 2) halts when no improvements can be found. The Route Addition Algorithm follows after the k-Split Interchange Algorithm has completed all the split interchanges which result in a positive distance saving.

Our Main Algorithm for constructing an SDVRP solution does not guarantee an optimal solution, but it is encouraging to notice that some characteristics of an optimal solution are guaranteed.

Theorem 5 : The SDVRP solution generated by the Main Algorithm does not contain a 2-split cycle.

Proof : The node interchange procedure insures this result by executing a *2 swap interchange* in case of 2-split cycle •

Assume a SDVRP problem in which each point's demand is identical and equal one unit. In this case, regardless of the spacial configuration of those demand points, the SDVRP solution and the VRP solution coincide. Another extreme scenario with

identical result is the case when the demand points require daily a full vehicle load. Again the SDVRP and the VRP solutions coincide.

Given a vehicle fleet with identical capacity $Q_v = Q$ for all v, and the demand $d_i = \frac{Q}{2} + \epsilon$ for i = 1,2,...,n ($\epsilon > 0$). Clearly, in this case the SDVRP solution and the VRP solution differ drastically in the number of required vehicles and in great likelihood also in the total distance traveled. The minimal number of vehicles in the SDVRP solution is clearly $\frac{n}{2} + 1$, whereas in the corresponding VRP solution this number is n. (This is clearly illustrated in Table II and in Table X in the Appendix for the parameter set [0.7 - 0.9].)

6. Computational Experiments

We attempt in our computational experiment to grasp the potential savings associated with the split demand delivery (VRP solution versus the SDVRP solution) as a function of the demand parameters. The spacial configuration and the number of demand points is kept fixed. The problem selected is the 75 point problem (Euclidien distances) originally presented in Eilon, Watson-Gandy, Christofides (1971) and repeated in many other computational experiments. (The 75 point coordinates are also given in Dror and Trudeau 1986.) In addition, assume a fleet of fixed vehicle capacity of 160 units each.

For the above problem configuration, 180 different problems were solved using six sets of parameters bounding the value of the point demand in units of vehicle capacity. The six parameter sets are:

(a) [0.01 - 0.1]

(b) [0.1 - 0.3]

(c) [0.1 - 0.5]

(d) [0.1 - 0.9]

(e) [0.3 - 0.7]

(f) [0.7 - 0.9]

The demands d_i (i = 1,2,...,75) were generated randomly from a uniform distribution for each parameter interval. For each of the 30 problems with the same parameters, different generating seeds were used, resulting in assignment of different demand values to the points for each problem.

Tables V - X in **Appendix 1** present the summery of the computational results. In **Table II** we highlight the percentage reduction in the total distance when allowing for split deliveries to the demand points. In Figure 4 we present a cumulative histogram for each set of 30 problems of the number of problems versus the % distance gain.

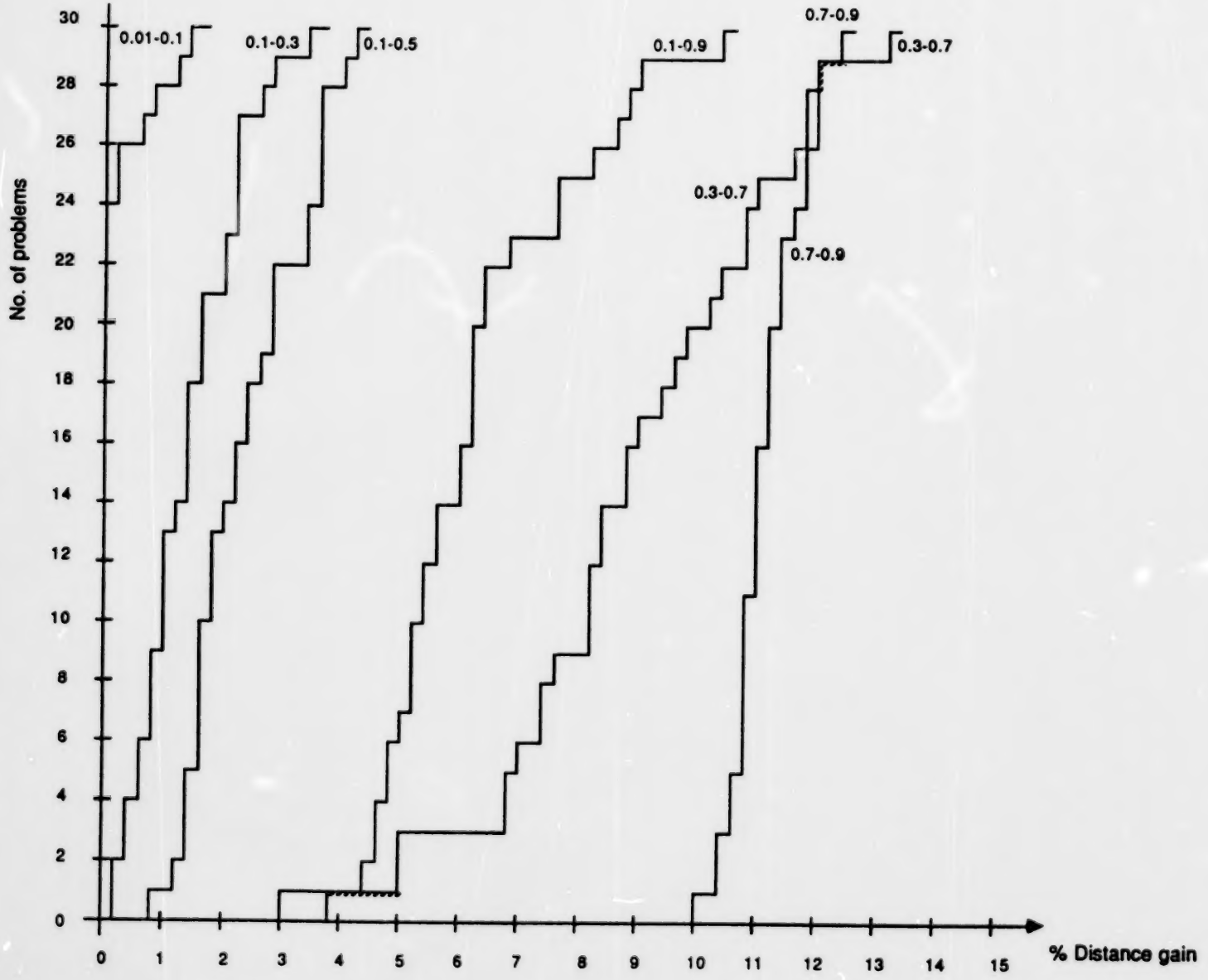

Figure 4: Cumulative histogram for the Number of problems versus the % Distance gain

	Problem Set [0.01 - 0.1]	Problem Set [0.1 - 0.3]	Problem Set [0.1 - 0.5]	Problem Set [0.1 - 0.9]	Problem Set [0.5 - 0.7]	Problem Set [0.7 - 0.9]
Average % Gain of Distance	0.22	1.50	2.68	6.20	9.01	11.24
No. of Sol. with Min.* No. of Veh.	30	21	20	11	10	9
Veh. Sav.	No. of Prob.	No. of Prob.	No. of Prob.	No. of Prob.	No. of Prob.	No. of Prob.
0	28	11				
1	2	19	10			
2			15			
3			5			
4				2		
5				6	1	
6				5	4	
7				9	6	
8				1	3	
9				3	7	
10	-			1	6	
11					3	
12						
13						4
14						20
15						6
Tot.	30	30	30	30	30	30

Table II : Summery of Computational Results.

Table II has two parts: In the first part we present the average % distance gain for the SDVRP solution over the VRP solution. For each set of demand parameters 30 problems were solved. Each problem was solved twice; once as a VRP and the second time as an SDVRP. The second row of Table II gives, for each demand parameter set, the number of problems which when solved as SDVRP used a minimal number of vehicles in their solution. This minimal number of vehicles is obtained by dividing the total demand by the capacity of the vehicle (we used a fixed vehicle capacity of 160 units). The second part of Table II gives an indication for the number of vehicles saved in the SDVRP solutions relative to the VRP solutions. For example for the 30 problems with point demand between 0.7Q and 0.9Q, though only 9 solutions had the minimal number of vehicles, in 4 solutions 13 vehicles were saved, in 20 solutions 14 vehicles were saved, and in 6 solutions 15 vehicles were saved. The average % distance gain for this set of problems was 11.24% with high at 12.56% and low at 10.02%. If the point demand is very low relative to the capacity of a vehicle, then there is no (or very little) advantage of solving the problem as SDVRP versus as VRP. This is clear from the column, in Table II, for the problem set of [0.01 - 0.1]. Once the individual point demands are above 0.1Q, then the advantage of the SDVRP solution versus VRP solution is obvious. For the second problem set (point demand between 0.1Q and 0.3Q), the individual demands are quite small in comparison to vehicle's capacity, yet the savings are significant. In 19 out of 30 problems one vehicle was saved and the average solution distance declined by 1.50% with high at 3.54% and low at 0.25%. In order to test whether the results (as measured by total distance only) for the SDVRP solutions are significantly different than the corresponding VRP solutions, a paired one-tailed t-test comparison of results was conducted. For the 6 problem sets these tests are summerized below in Table III.

Problem Parameter Set	$\alpha = 0.001$ T - Value	$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 > \mu_2$
[0.01 - 0.1]	2.06	accepted
[0.1 - 0.3]	10.19	rejected
[0.1 - 0.9]	17.91	rejected
[0.1 - 0.5]	14.04	rejected
[0.3 - 0.7]	19.16	rejected
[0.7 - 0.9]	112.26	rejected

Table III : Paired one-tailed t-tests of SDVRP versus VRP solutions (n = 30).

As expected, for the very small point demands there is no significant difference in performance of the SDVRP solutions and the VRP solutions at $\alpha = 0.001$. But even for the parameter set [0.01 - 0.1], rising the α level to 0.05 would force the rejection of the hypothesis that the VRP and the SDVRP perform about the same. For all other problem sets, this performance difference is highly significant.

The routes constructed by the Main Algorithm do not contain 2-split cycles. This result follows from Theorem 5. Though k-split cycles ($k \geq 3$) can in principle occur in the SDVRP solutions, for all the 180 problems solved in this study, not one occurance of k-split cycle was registered. This empirical computational result reasserts the authors belief in the efficiency of this routing scheme.

One additional important factor when comparing two computerized solution schemes for a problems of this nature, is the computational effort as measured by seconds of CPU time on a given machine. Table IV provides this comparison for SDVRP and VRP procedures observed on CYBER 173 using FTN5 compliler.

Problem Set	SDVRP Solution (average CPU sec.)	VRP Solution (average CPU sec.)
[0.01 - 0.1]	28.8	25.2
[0.1 - 0.3]	33.6	12.4
[0.1 - 0.9]	86.1	8.3
[0.1 - 0.5]	46.9	9.1
[0.3 - 0.7]	86.0	7.8
[0.7 - 0.9]	222.2	9.9

Table IV : Average CPU time (in seconds) on CYBER 173 with FTN5 compiler.

The CPU times for the SDVRP procedure present the combined time for the two stages of the Main Algorithm. In order to obtain the average CPU time for Stage 2 alone one has to subtract the two columns in Table IV. Our comment about the CPU times presented in Table IV is that these numbers do not represent any effort to streamline the procedures presented in this paper. The numbers only reflect an initial attempt to construct a computerized solution schemes for the SDVRP and the VRP. A streamlined professional software package of the above procedures would very likely substentially reduce the numbers in Table IV.

7. Summery and Conclusions

This paper examines a variation of the generic vehicle routing problem for which deliveries to a single point are allowed to be split between a number of vehicles. Though it represents a relaxation of the generic VRP, this problem is still in the NP-complete class of problems. Since presently only small size VRP problems can be solved to optimality, a heuristic approaches to the VRP and to our problem are in order. Our search for solution schemes concentrates on intuitively simple point interchange concepts. After gaining some understanding of the optimal solution properties, two new heuristic procedures are developed on top of some better known and succesfull VRP procedures. The two new procedures exploit the relaxation allowing for split deliveries. The first procedure attempts to split the demand to be delivered to a point between a number of vehicles. The second procedure examines whether elimination of split deliveries by adding a route can benefit the final route structure. When a final solution procedure is constructed from initial route building and route improvement schemes together with the new split delivery and route addition procedures, a powerfull route construction algorithm emerges. It is clearly demonstrated (See Figure 4, Table II, and the more detailed tables in the Appendix.) that allowing for split deliveries in a single commodity environment can result in very significant cost savings.

On a close examination of the routes constructed by our procedure, we noticed few interesting characteristics. The procedure constructs routes which consist of demand points from the same geographic area. It 'cleans' geographic zones. The split demand points are usually the one with above average demand. The so called splits which result in high distance savings occour relatively further from the depot. The resulting route structure reveals that the 'routing decisions' implemented by this route construction algorithm are intuitively 'correct' with the added feature of being able to examine the entire complex routing situation and not just small part of it.

Routing procedures which make a 'cost' sense and construct routes which make dispatching sense are the ones with the highest hope of being ultimately implemeted in practice.

References

Bodin, L., Golden, B., Assad, A., and Ball, M., 'The state of the art in the routing and scheduling of vehicles and crews', **Computers and Operations Research**, Special issue, June 1983.

Christofides, N., 'Vehicle routing', **The Traveling Salesman Problem**, (Edited by E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan, and D.B. Shmoys), John Wiley and Sons, Ltd. (1985), pp.,431-44.

Christofides, N., Mingozzi, A., Toth, P., 'Exact algorithms for the vehicle routing problem based on spanning tree and shortest path relaxations', **Mathematical Programming**,20,(1980),pp.,255-282.

Christofides, N., Mingozzi, A., Toth, P., 'State-space relaxation procedures for the computation of bounds to routing problems', **Networks**, 11,(1981),pp.,145-164.

Dror, M., and Trudeau, P., 'Stochastic vehicle routing with modified savings algorithm', **European Journal of Operational Research**,23,2, (1986),pp.,228-235.

Dror, M., and Levy, L., 'A vehicle routing improvement algorithm: comparison of 'greedy' and a 'matching' implementation for inventory routing', **Computers and Operations Research**,13,1,(1986),pp.,33-45.

Dror, M., Ball, M., and Golden, B., 'A computational comparison of algorithms for inventory routing problem', **Annals of Operations Research** ,4,(1985/6),pp.,3-23.

Eilon, S., Watson-Gandy, C.D.T., Christofides, N., 'Distribution Management: Mathematical Modeling and Practical Analysis', **Griffin**, London, (1971).

Fisher, M., 'The Lagrangian method for solving integer programming problems', **Management Science**, 27,(1981), pp.,1-18.

Fisher, M., Jaikumar, R., 'A generalized assignment heuristic for vehicle routing', **Networks**,11,(1981),pp.,109-124.

Garey,M.R., Johnson, D.S., 'Computers and Intractability: A Guide to the Theory of NP-Completeness', **Freeman**,San Francisco,(1979).

Gavish, B., and Graves, S.C., 'Scheduling in transportation and distribution systems: formulations and new relaxations', Working paper, Graduate School of Management, University of Rochester,(1982).

Golden, B., Magnanti, Nguyen, H.Q., 'Implementing vehicle routing algorithms', **Networks**, 7, (1977), pp.,113-148.

Lenstra, J.K., Rinnooy Kan, A.H.G., 'Complexity of vehicle routing and scheduling problems', **Networks**,11,(1981),pp.,221-227.

Laporte, G., Norbert, Y., Desrochers, M., 'Optimal routing under capacity and Distance restrictions', **Operations Research**,33,5,(1985), pp., 1050-1073

Laporte, G., Mercure, H., Norbert, Y., 'An exact algorithm for the asymmetric capacitated vehicle routing problem', **Networks**,16,(1986),pp., 33-46.

Interval [0.01 - 0.1]		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Gain
1	581	652.0	4	652.0	4	-
2	651	663.2	5	663.2	5	-
3	595	643.9	4	643.9	4	-
4	599	638.8	4	638.8	4	-
5	610	643.4	4	643.4	4	-
6	653	659.9	5	659.9	5	-
7	601	650.7	4	650.7	4	-
8	621	637.4	4	637.4	4	-
9	629	633.6	4	633.6	4	-
10	540	622.0	4	622.0	4	-
11	590	662.4	4	662.4	4	-
12	613	634.0	4	632.9	4	0.16
13	645	637.1	5	637.0	5	0.01
14	667	666.9	5	666.9	5	-
15	659	641.4	5	637.5	5	0.60
16	569	656.6	4	656.6	4	-
17	548	635.8	4	635.8	4	-
18	639	640.4	5	637.4	4	0.48
19	700	701.8	5	701.8	5	-
20	651	645.2	5	645.2	5	-
21	611	642.1	4	642.1	4	-
22	615	619.7	4	619.7	4	-
23	566	643.5	4	643.5	4	-
24	629	657.7	4	657.7	4	-
25	680	645.8	5	645.8	5	-
26	623	665.6	4	658.3	4	1.09
27	656	671.7	5	671.7	5	-
28	640	668.6	5	660.3	4	1.24
29	658	667.8	5	667.8	5	-
30	565	632.1	4	632.1	4	-

Table V : Computational results for demand parameters [0.01 - 0.1].

Interval [0.1 - 0.3]		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Gain
1	2524	1175.6	17	1165.9	17	0.83
2	2306	1107.3	16	1090.1	15	1.55
3	2201	1088.1	15	1065.1	14	2.11
4	2452	1157.4	17	1116.4	16	3.54
5	2378	1148.5	16	1140.0	16	0.74
6	2475	1204.2	17	1197.1	17	0.59
7	2277	1109.2	16	1089.7	15	1.76
8	2512	1194.7	17	1168.2	16	2.22
9	2419	1166.6	17	1148.8	16	1.53
10	2355	1098.2	16	1088.7	15	0.87
11	2348	1122.3	16	1118.4	15	0.35
12	2343	1124.8	16	1113.1	16	1.04
13	2315	1128.2	15	1125.4	15	0.25
14	2342	1121.7	16	1116.1	16	0.50
15	2456	1170.7	17	1153.5	16	1.47
16	2358	1138.2	16	1108.1	15	2.64
17	2287	1110.8	15	1097.6	15	1.19
18	2248	1129.6	16	1106.7	15	2.03
19	2240	1108.2	15	1088.8	15	1.75
20	2476	1157.4	17	1147.4	16	0.86
21	2446	1186.7	17	1158.5	16	2.37
22	2521	1203.9	17	1169.7	16	2.84
23	2304	1114.2	16	1088.6	15	2.30
24	2335	1126.0	16	1112.1	15	1.23
25	2348	1137.4	16	1124.6	15	1.12
26	2367	1136.9	16	1111.5	15	2.23
27	2391	1175.0	16	1155.1	16	1.69
28	2465	1183.6	17	1174.3	16	0.79
29	2381	1143.7	16	1125.7	16	1.58
30	2331	1110.6	16	1098.4	16	1.10

Table VI : Computational results for demand parameters [0.1 - 0.3].

Interval [0.1 - 0.5]		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Ga
1	3712	1600.7	26	1563.7	24	2.31
2	3420	1482.1	23	1457.1	22	1.69
3	3474	1490.7	24	1429.4	22	4.11
4	3666	1610.4	26	1581.5	24	1.79
5	3681	1542.9	25	1510.2	24	2.12
6	3635	1578.6	26	1512.4	24	4.20
7	3586	1519.8	25	1462.8	23	3.75
8	3759	1620.3	26	1579.6	24	2.51
9	3490	1513.2	24	1479.5	23	2.23
10	3381	1443.2	24	1406.9	22	2.51
11	3643	1542.3	25	1516.0	23	1.70
12	3734	1541.2	26	1510.9	24	1.97
13	3651	1502.7	25	1482.7	24	1.33
14	3712	1529.6	25	1516.9	24	0.84
15	3856	1612.6	28	1552.2	25	3.75
16	3523	1552.1	25	1505.9	23	2.98
17	3621	1563.9	26	1518.6	23	2.90
18	3351	1457.1	23	1434.7	22	1.54
19	3838	1588.9	27	1560.0	25	1.82
20	3351	1443.2	23	1417.0	22	1.81
21	3902	1641.7	28	1581.1	25	3.69
22	3286	1413.7	23	1391.6	22	1.56
23	3528	1499.1	25	1443.9	23	3.68
24	3608	1543.7	26	1501.2	23	2.76
25	3642	1555.2	25	1500.9	23	3.49
26	3513	1497.5	24	1474.8	23	1.52
27	3595	1521.6	26	1468.8	23	3.47
28	3597	1526.9	24	1501.8	23	1.65
29	3597	1498.0	25	1472.5	23	1.71
30	3652	1533.3	26	1489.1	24	2.88

Table VII : Computational results for demand parameters [0.1 - 0.5].

Interval 0.1 - 0.9		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Gain
1	6354	2526.2	47	2365.5	41	6.36
2	5786	2311.2	43	2182.5	38	5.57
3	6500	2592.8	51	2320.4	42	10.51
4	6193	2360.2	45	2243.1	40	4.96
5	5373	2185.6	39	2087.2	35	4.50
6	5567	2202.3	41	2092.7	36	4.98
7	5634	2288.4	42	2170.7	36	5.14
8	5879	2328.1	44	2200.5	37	5.48
9	5718	2252.4	43	2122.8	37	5.75
10	5797	2306.0	44	2148.0	37	6.85
11	5818	2379.8	43	2241.9	37	5.79
12	6091	2446.2	46	2294.2	39	6.21
13	5782	2448.6	44	2261.3	37	7.65
14	6430	2557.3	50	2336.2	41	8.65
15	6759	2736.3	53	2492.8	43	8.90
16	6116	2390.2	46	2243.1	40	6.15
17	5946	2399.5	45	2248.4	38	6.29
18	5503	2157.2	40	2041.9	36	5.35
19	5812	2295.3	43	2187.0	37	4.72
20	6164	2529.6	49	2298.0	40	9.16
21	5407	2201.7	39	2131.9	34	3.17
22	6013	2388.5	46	2203.8	38	7.73
23	5673	2236.7	41	2118.0	36	5.31
24	5781	2271.6	44	2150.1	37	5.35
25	5839	2324.1	45	2172.6	38	6.52
26	6045	2412.1	47	2213.5	40	8.23
27	5681	2265.3	43	2125.1	37	6.19
28	5905	2343.6	44	2190.2	38	6.55
29	5919	2343.3	43	2233.5	38	4.69
30	6152	2415.0	46	2263.6	39	6.27

Table VIII : Computational results for demand parameters [0.1 - 0.9].

Interval [0.3 - 0.7		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Gain
1	5870	2396.0	46	2137.4	37	10.80
2	5883	2410.4	46	2176.3	38	9.71
3	5736	2335.3	43	2173.8	37	6.92
4	5729	2311.4	44	2146.3	37	7.14
5	5921	2471.3	45	2259.5	38	8.57
6	6040	2472.9	49	2247.2	39	9.13
7	5864	2343.3	44	2167.9	37	7.48
8	6027	2542.9	48	2267.7	39	10.82
9	6260	2521.4	50	2258.8	40	10.41
10	5904	2411.8	45	2210.3	38	8.35
11	6384	2738.5	51	2418.8	41	11.68
12	5622	2247.0	41	2161.0	36	3.83
13	6006	2521.0	48	2216.4	39	12.08
14	6187	2601.0	49	2330.7	39	10.39
15	5943	2461.5	47	2254.9	38	8.39
16	6112	2649.3	50	2326.3	39	12.19
17	5886	2415.7	44	2236.5	37	7.42
18	5932	2519.9	45	2282.2	38	9.43
19	6004	2481.7	48	2262.8	39	8.82
20	5827	2264.1	43	2146.9	37	5.18
21	5758	2331.4	43	2210.7	37	5.18
22	6301	2585.5	50	2331.4	40	9.83
23	6076	2592.3	50	2281.1	39	12.00
24	5800	2400.2	43	2236.8	37	6.81
25	5957	2478.1	46	2256.1	38	8.96
26	5952	2392.1	47	2186.8	38	8.58
27	6090	2563.6	49	2280.2	39	11.05
28	6127	2627.1	50	2279.1	39	13.25
29	6039	2551.6	47	2357.4	38	7.61
30	5938	2436.5	46	2232.2	38	8.38

Table IX : Computational results for demand parameters [0.3 - 0.7].

Interval 0.7 - 0.9		VRP Solution		SDVRP Solution		
Prob. No.	Tot. Demand	Tot. Dist.	No. of Veh.	Tot. Dist.	No. of Veh.	Percentage Gain
1	9493	3630.9	75	3197.7	61	11.93
2	9554	3630.9	75	3216.3	61	11.42
3	9554	3630.9	75	3218.1	61	11.37
4	9442	3630.9	75	3192.1	60	12.08
5	9556	3630.9	75	3234.7	61	10.91
6	9466	3630.9	75	3199.7	61	11.88
7	9472	3630.9	75	3199.0	60	11.89
8	9609	3630.9	75	3228.6	61	11.08
9	9581	3630.9	75	3237.8	61	10.83
10	9650	3630.9	75	3218.9	62	11.35
11	9605	3630.9	75	3244.3	61	10.65
12	9562	3630.9	75	3228.4	61	11.09
13	9530	3630.9	75	3213.1	61	11.51
14	9566	3630.9	75	3235.6	61	10.89
15	9574	3630.9	75	3251.6	61	10.44
16	9486	3630.9	75	3203.4	60	11.77
17	9638	3630.9	75	3247.1	61	10.57
18	9592	3630.9	75	3228.7	61	11.08
19	9619	3630.9	75	3239.0	61	10.79
20	9535	3630.9	75	3217.6	61	11.38
21	9553	3630.9	75	3218.6	60	11.36
22	9636	3630.9	75	3226.3	62	11.14
23	9641	3630.9	75	3237.9	61	10.82
24	9439	3630.9	75	3202.0	60	11.81
25	9626	3630.9	75	3236.9	62	10.85
26	9535	3630.9	75	3212.1	61	11.53
27	9745	3630.9	75	3267.1	62	10.02
28	9415	3630.9	75	3174.7	60	12.56
29	9575	3630.9	75	3232.5	61	10.97
30	9600	3630.9	75	3231.5	61	11.00

Table X : Computational results for demand parameters [0.7 - 0.9].